

Truthful Elicitation of Imprecise Forecasts

Anurag Singh¹, Siu Lun Chau² and Krikamol Muandet¹

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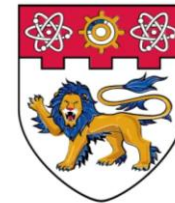
²Nanyang Technological University, Singapore



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Rational Intelligence Lab in CISPA



Krikamol Muandet
(PI)



Siu Lun Chau
Moved to NTU and recruiting!

Rational Intelligence Lab

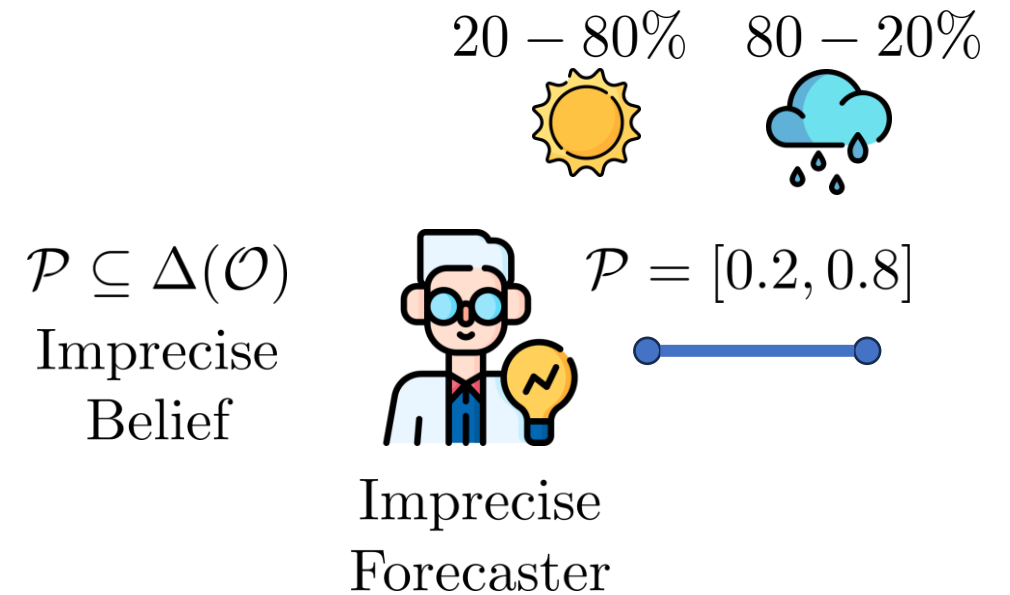
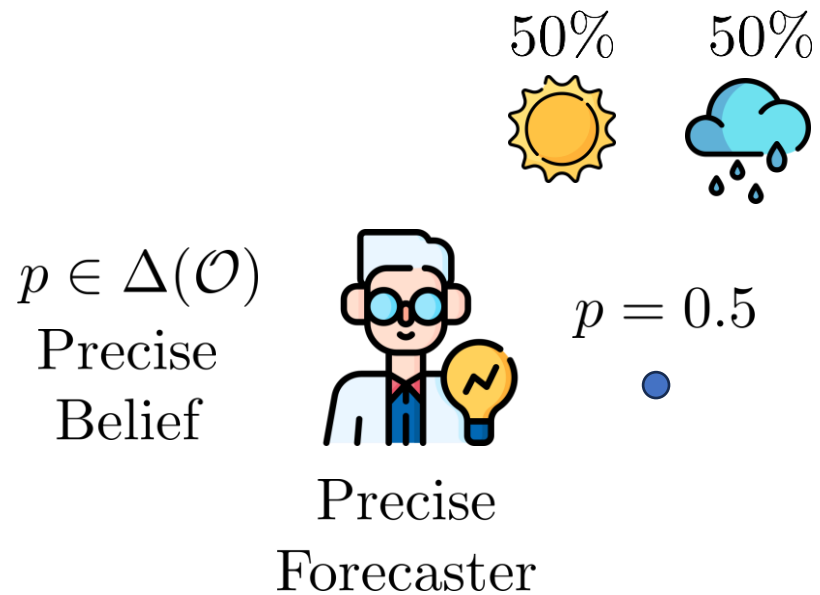
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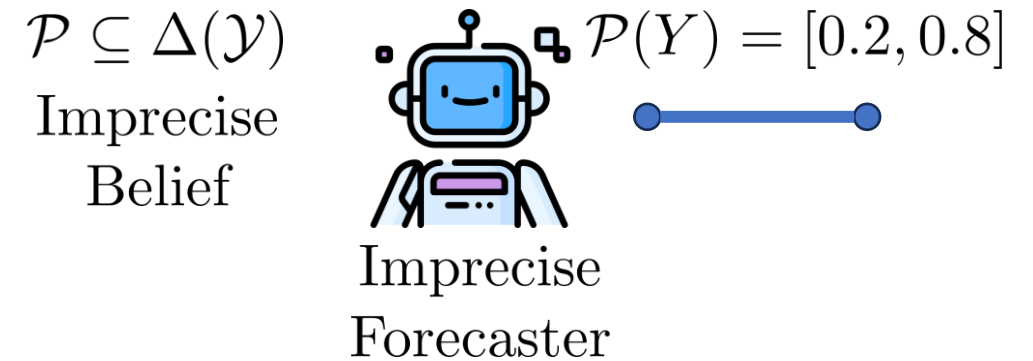
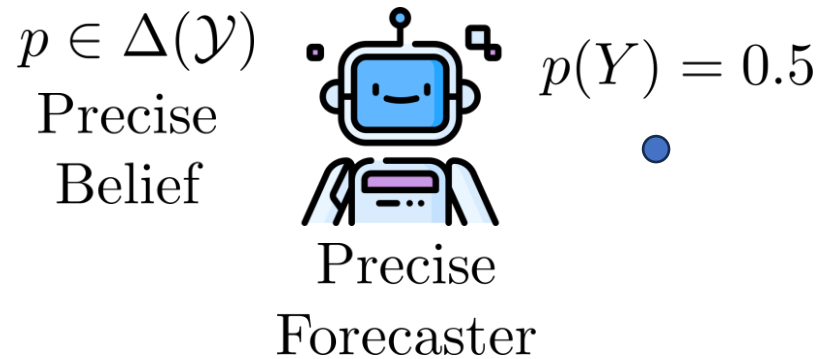
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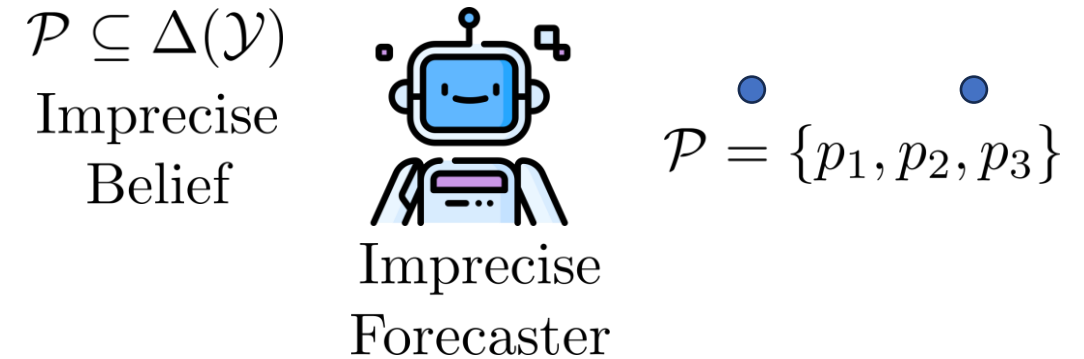
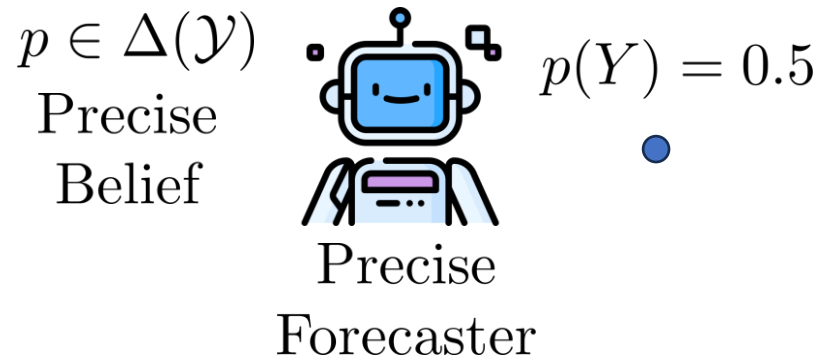
What is an imprecise forecaster?



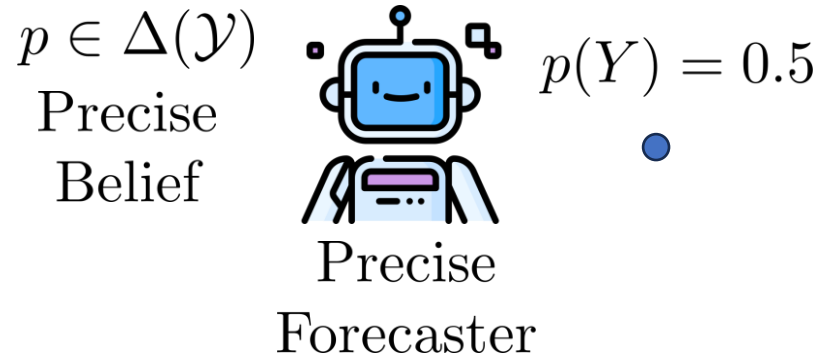
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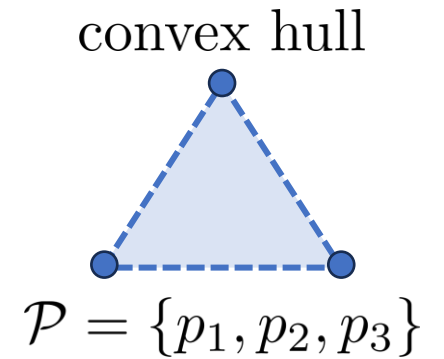
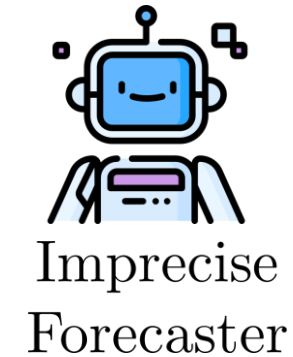
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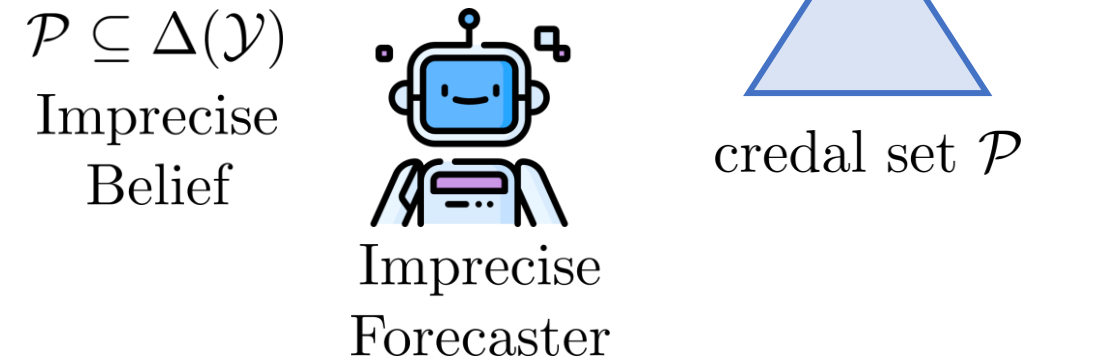
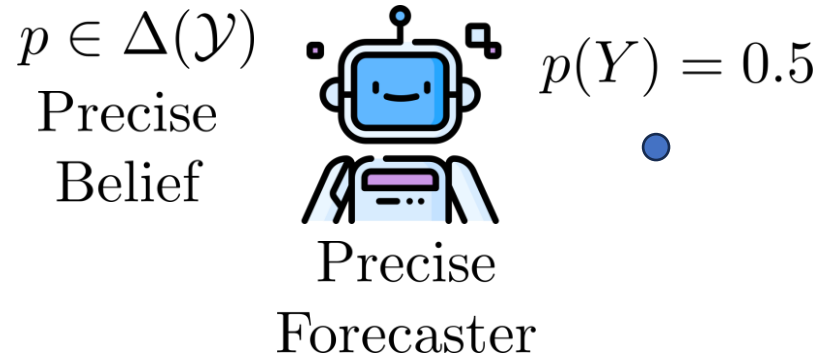
What is an imprecise forecaster?



$\mathcal{P} \subseteq \Delta(\mathcal{Y})$
Imprecise
Belief



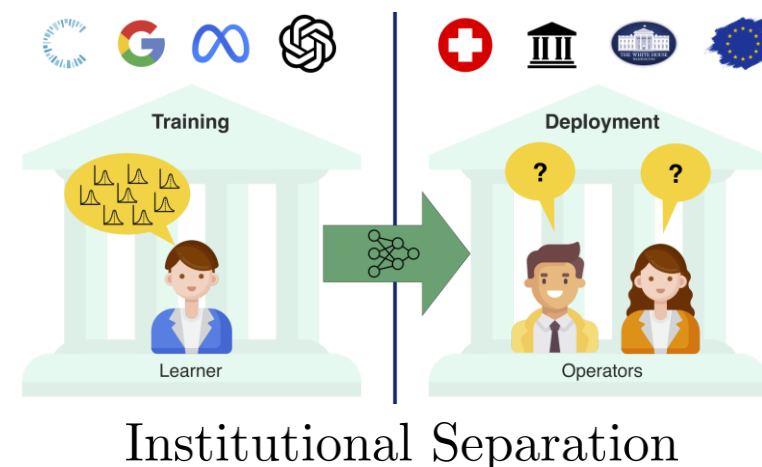
Credal set of an imprecise forecast



Why do we care about credal sets?

Can we train predictors that generalise for all downstream decision makers ?

Yes, with a credal set of models.



Domain Generalisation via Imprecise Learning

Anurag Singh¹ Siu Lun Chau¹ Shahine Bouabid² Krikamol Muandet¹

Abstract

Out-of-distribution (OOD) generalisation is challenging because it involves not only learning from empirical data, but also deciding among various notions of generalisation, e.g., optimising the average case risk, worst case risk, or interpolating

(LLM) that surpass human-level generalisation capabilities in specific domains.

Despite notable achievements, these systems may catastrophically fail when operated on out-of-domain (OOD) data because theoretical guarantees for their generalisation hinge on the assumption of independent and identically dis-



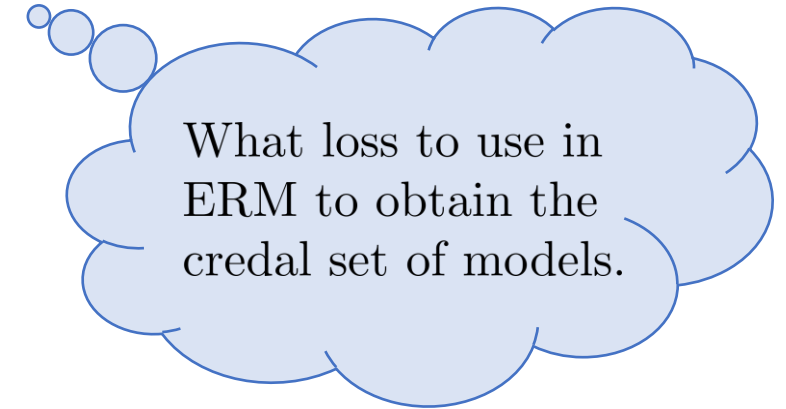
Appeared in ICML 2024
as spotlight

Abstracting the same question

Wait....

Was this not impossible?

The impossibility is only for the
real-valued scoring rules! Ours is
random



Truthful Elicitation of Imprecise Forecasts

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Abstract

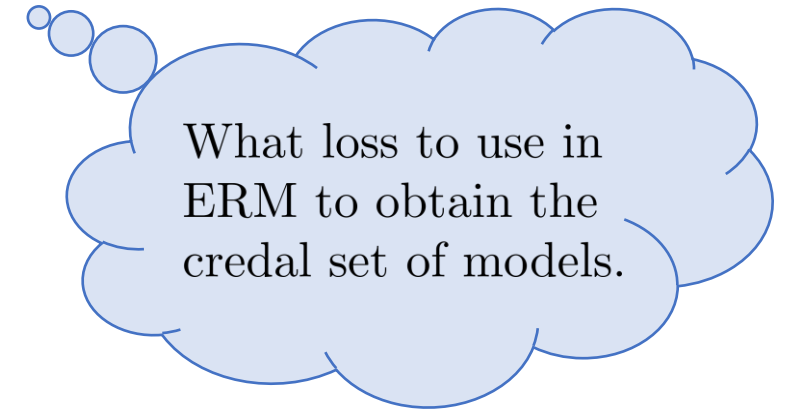
ing and Raftery, [2007]. They also serve as a backbone for eliciting other distributional properties such as their mo-

Abstracting the same question




Wait....

Was this not impossible?

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real-valued scoring rules! Ours is
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Our Contributions:

-  Formalise the role of decision maker in imprecise forecast elicitation.
-  Circumvent prior impossibility results to propose a **strictly proper** randomised scoring rule!
-  Bonus: Connection to social choice theory

Truthful Elicitation of Imprecise Forecasts

Anurag Singh¹

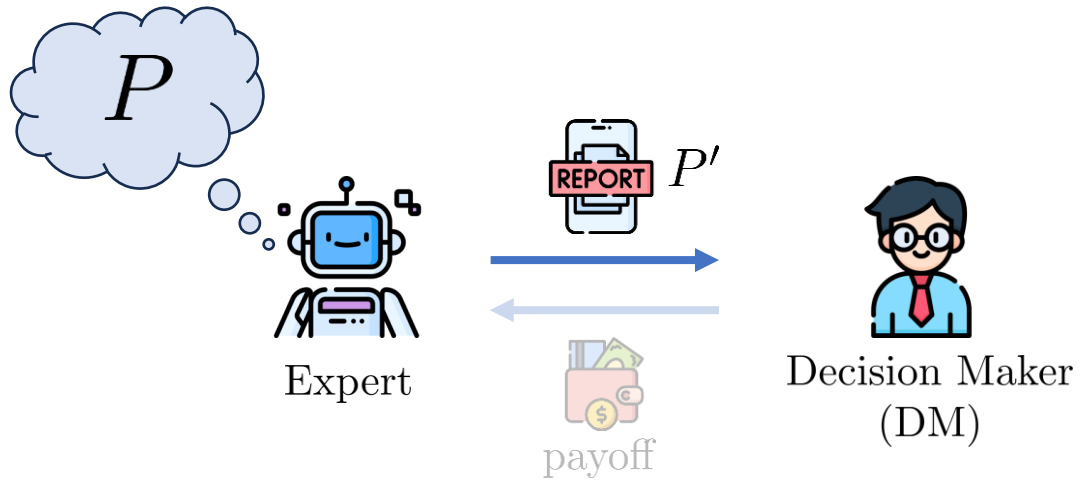
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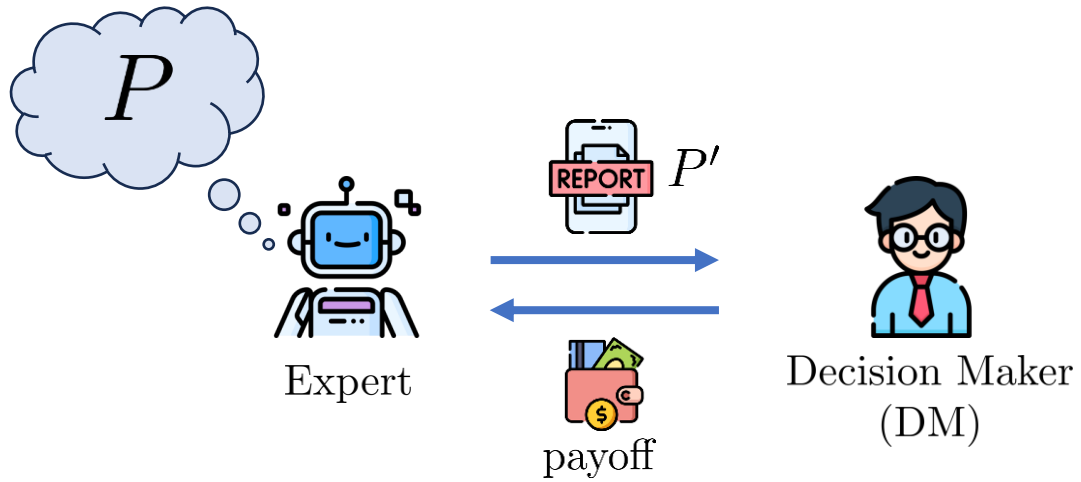
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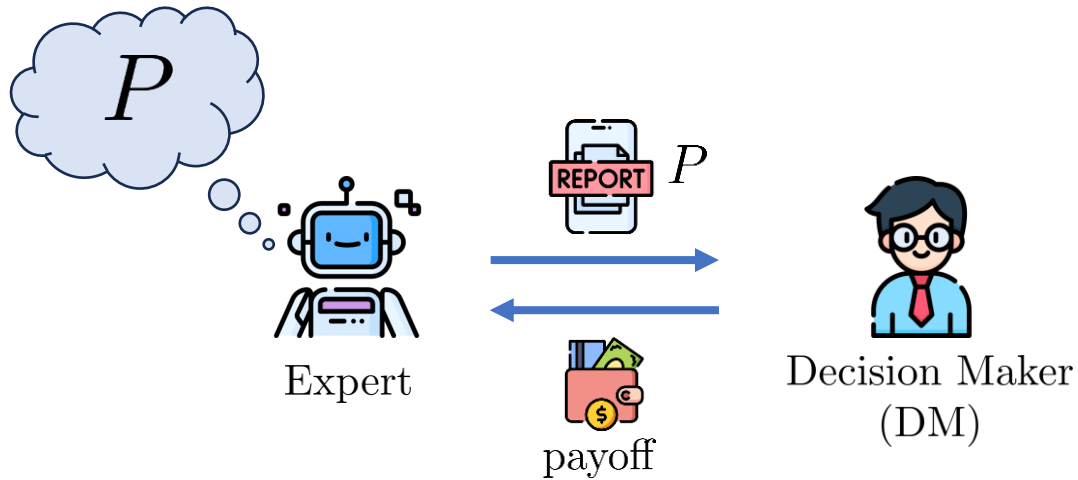
What is Elicitation



What is Elicitation



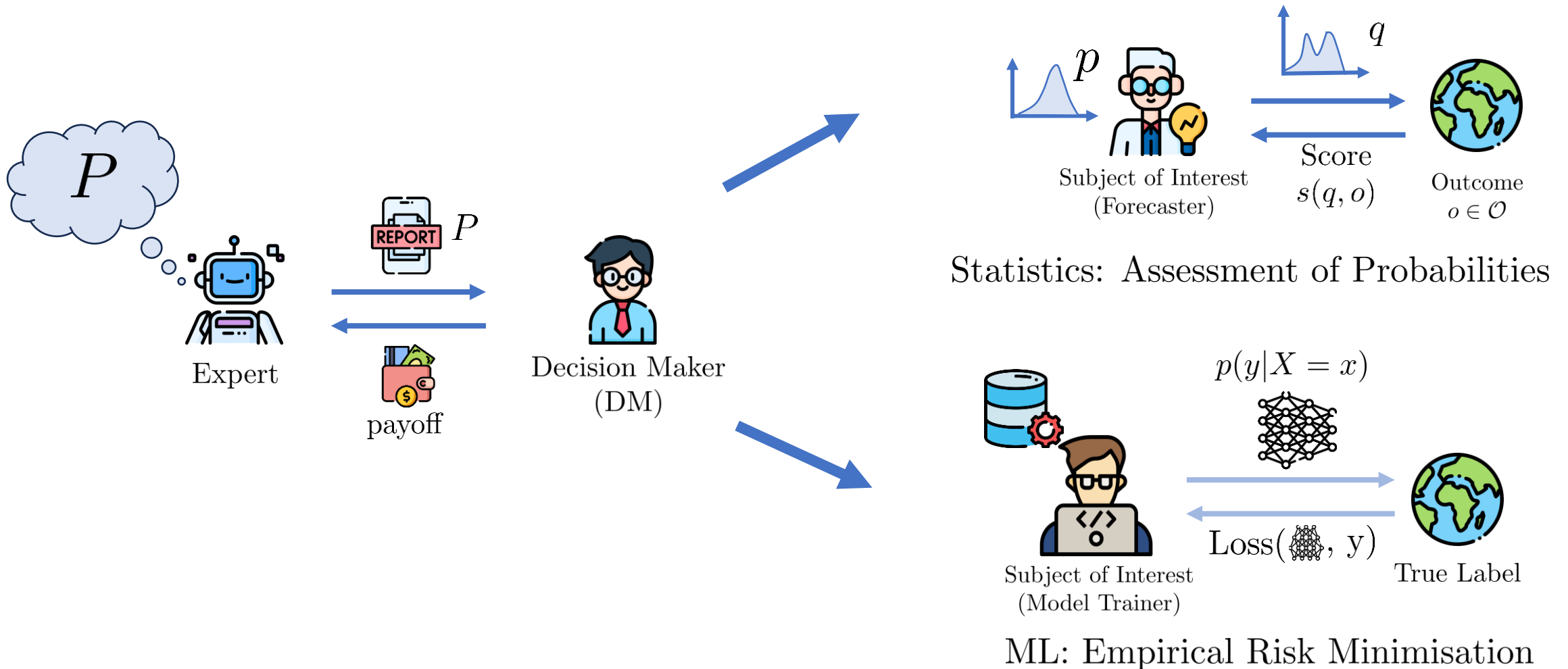
What is Elicitation



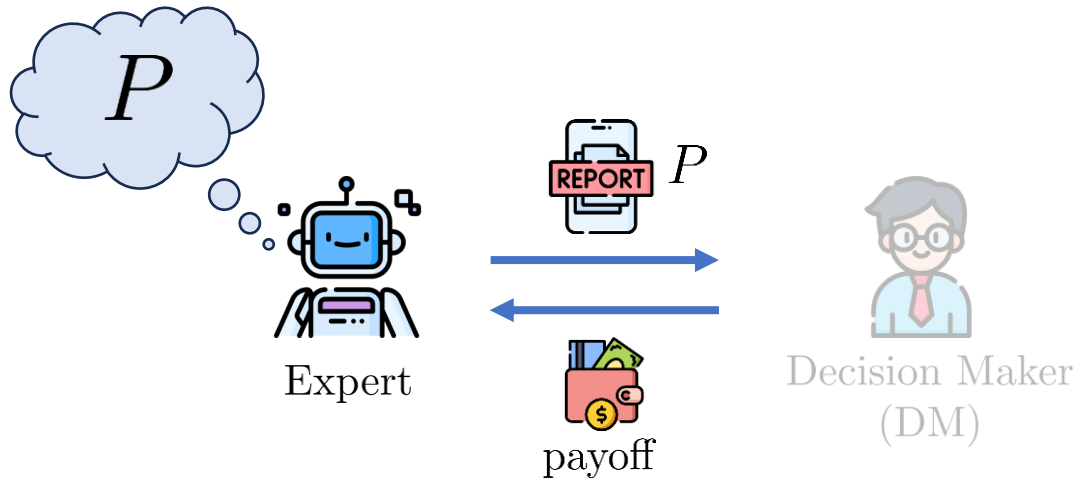
A payoff mechanism performs **truthful** elicitation if it can incentivise the expert to report their true belief.

... In other words, speaking truth is dominant strategy for expert.

Applications of truthful elicitation

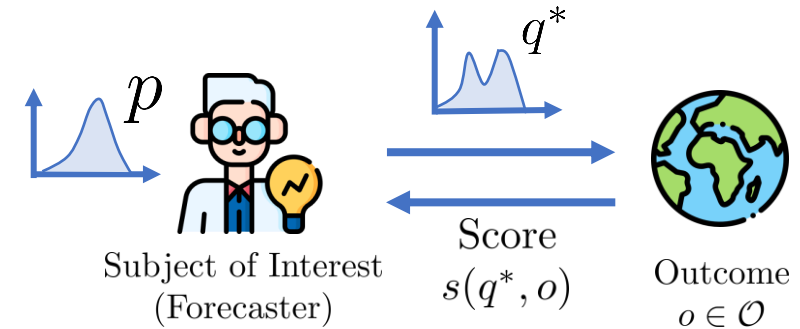


Truthful elicitation of precise forecasts



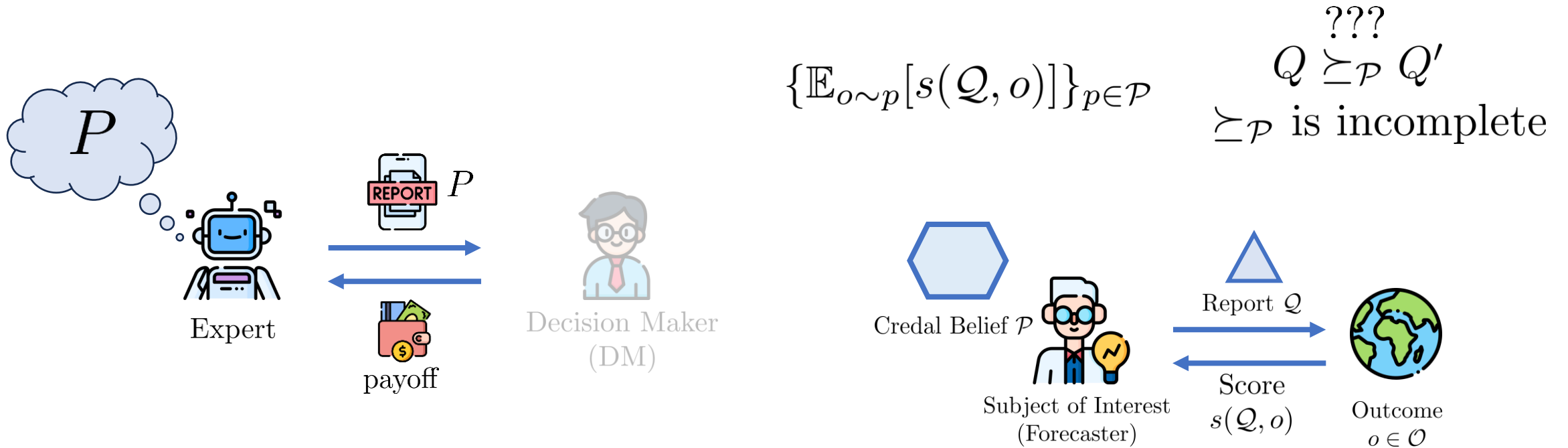
$$\mathbb{E}_{o \sim p}[s(q^*, o)] \quad q^* \succeq_p q_2 \succeq_p \dots$$

\succeq_p is complete

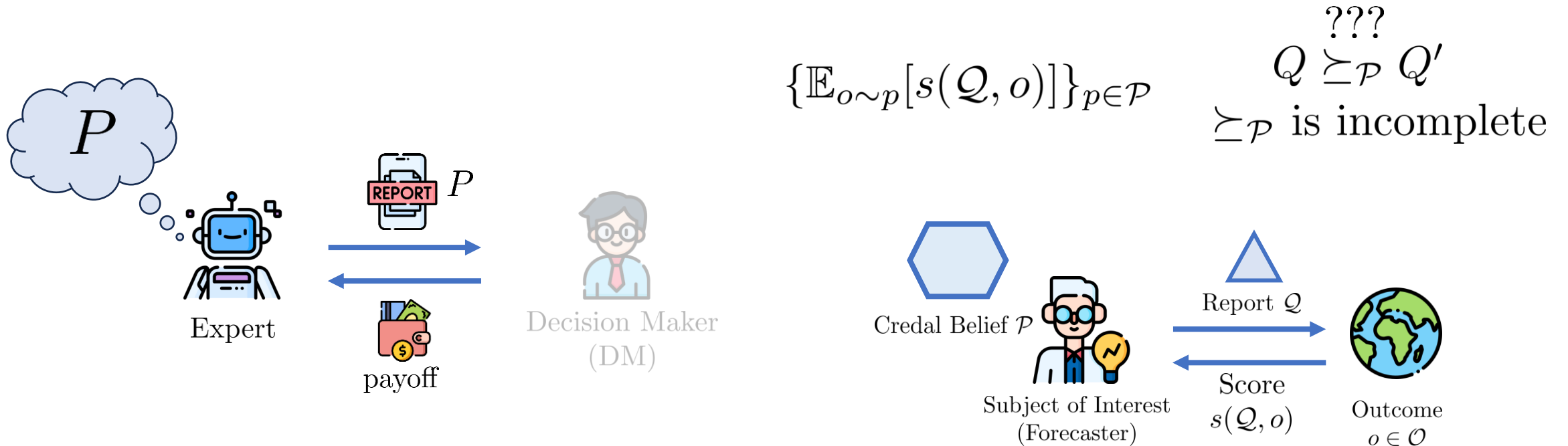


Elicitation of precise forecasts ignores the DM!

Truthful elicitation of imprecise forecasts

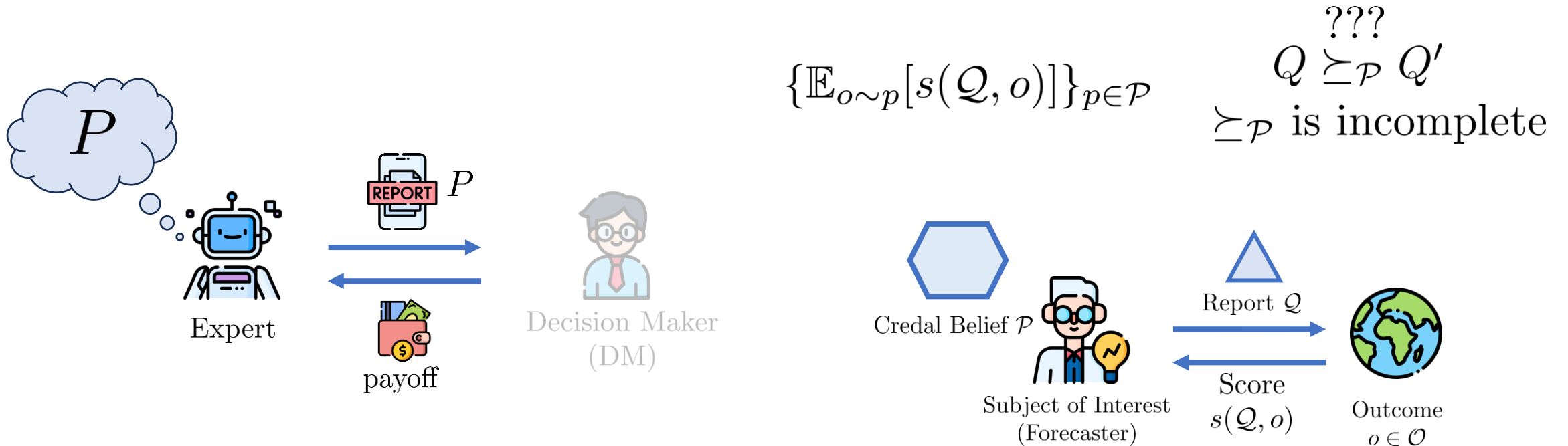


Truthful elicitation of imprecise forecasts



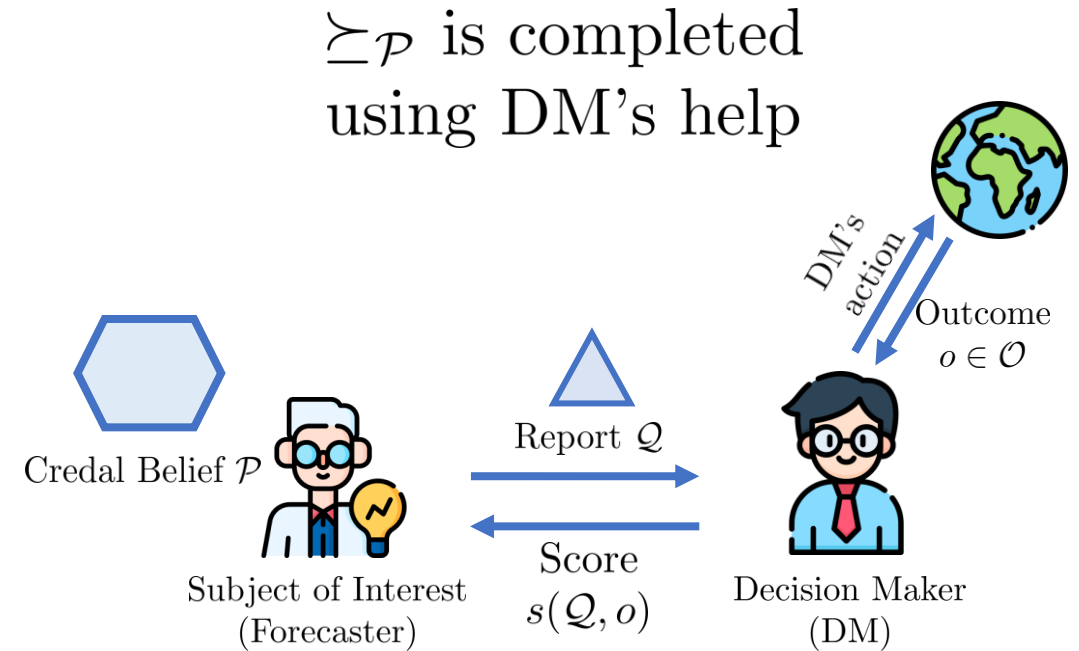
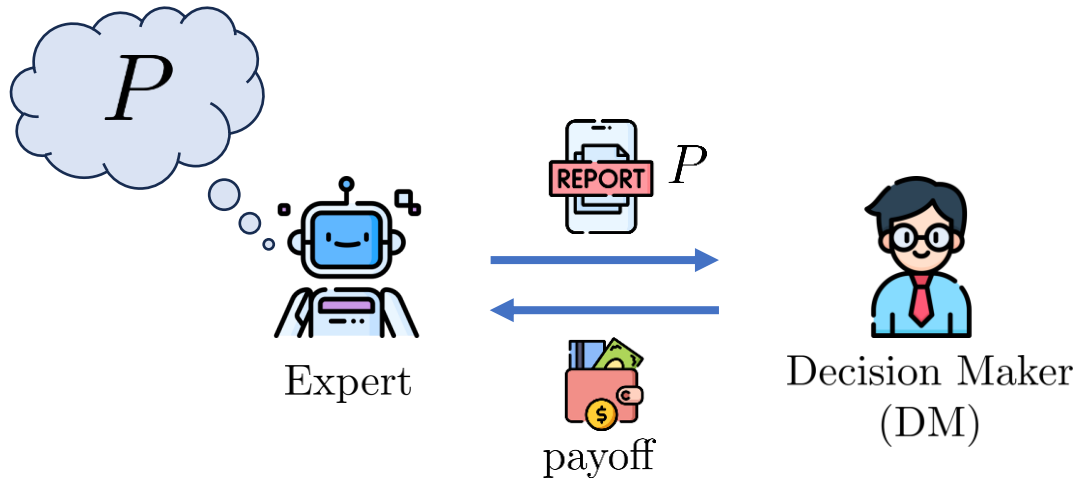
Impossibility results on IP scoring rules
(Seidenfeld 2012, Mayo-Wilson 2015 and Schoenfeld, 2017)

Truthful elicitation of imprecise forecasts

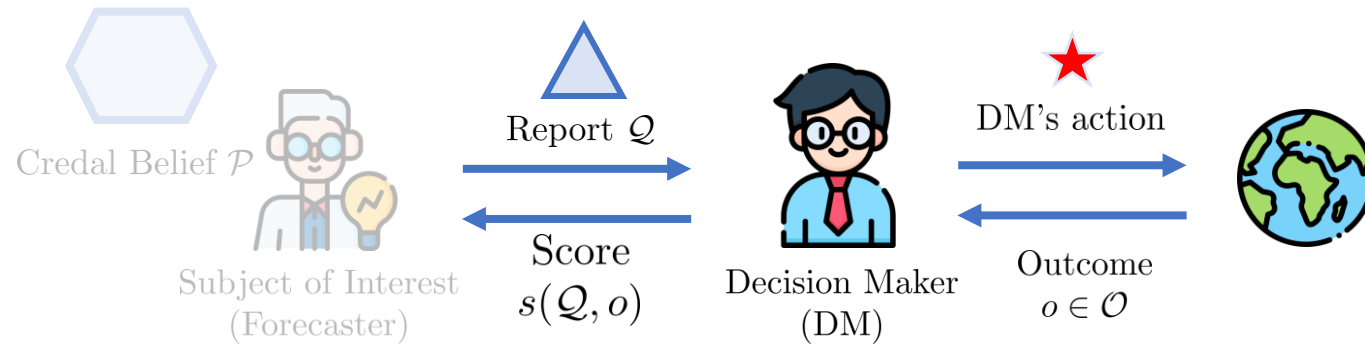


For any belief $\mathcal{P} \in 2^{\Delta(\mathcal{O})}$ the only proper scoring rule $s : 2^{\mathcal{O}} \times \mathcal{O} \rightarrow \mathbb{R}$ is for a $k \in \mathbb{R}$, $s(Q, o) = k$ for all $Q \in 2^{\Delta(\mathcal{O})}$

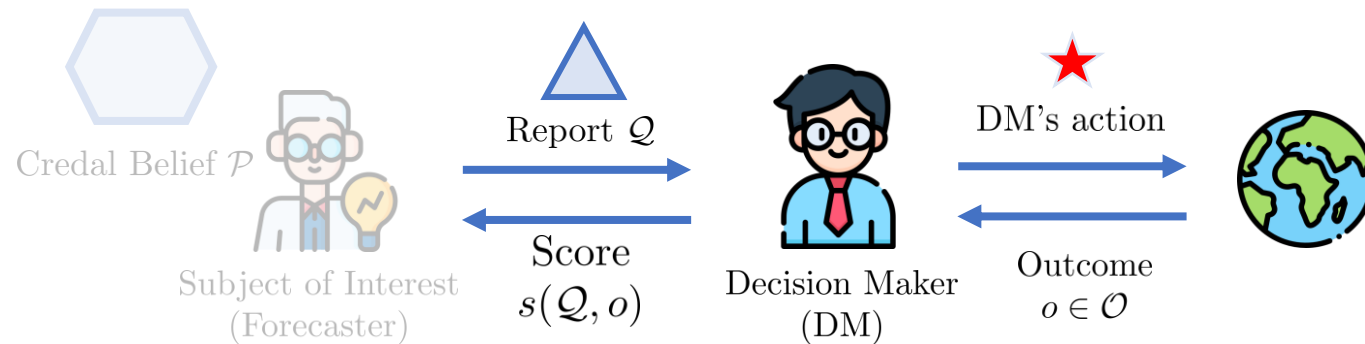
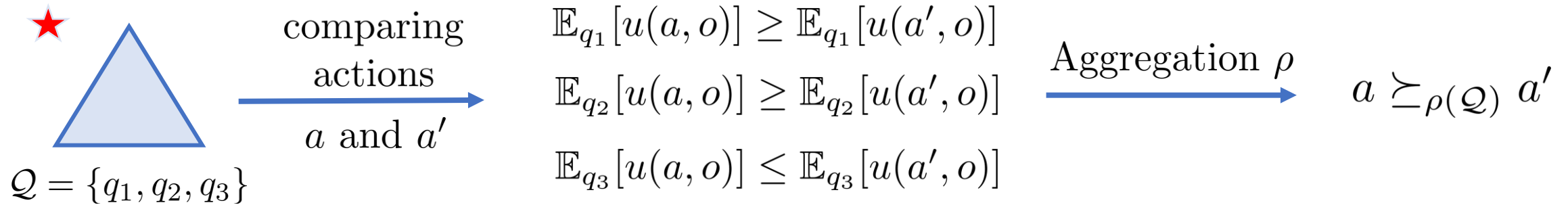
Truthful elicitation of imprecise forecasts



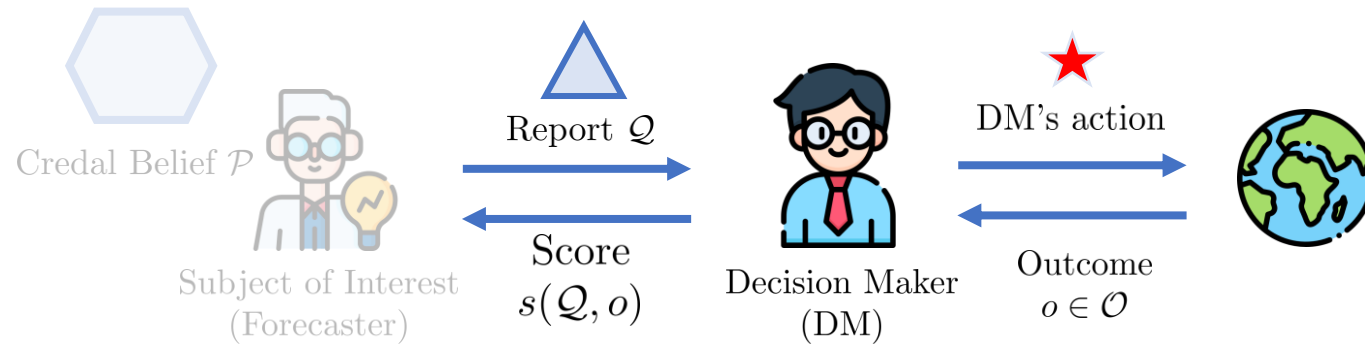
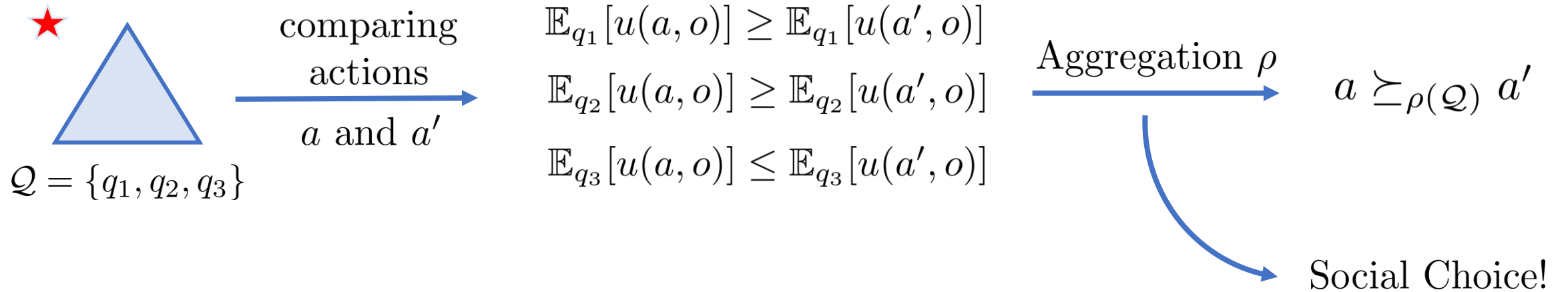
What is the exact form of this help?



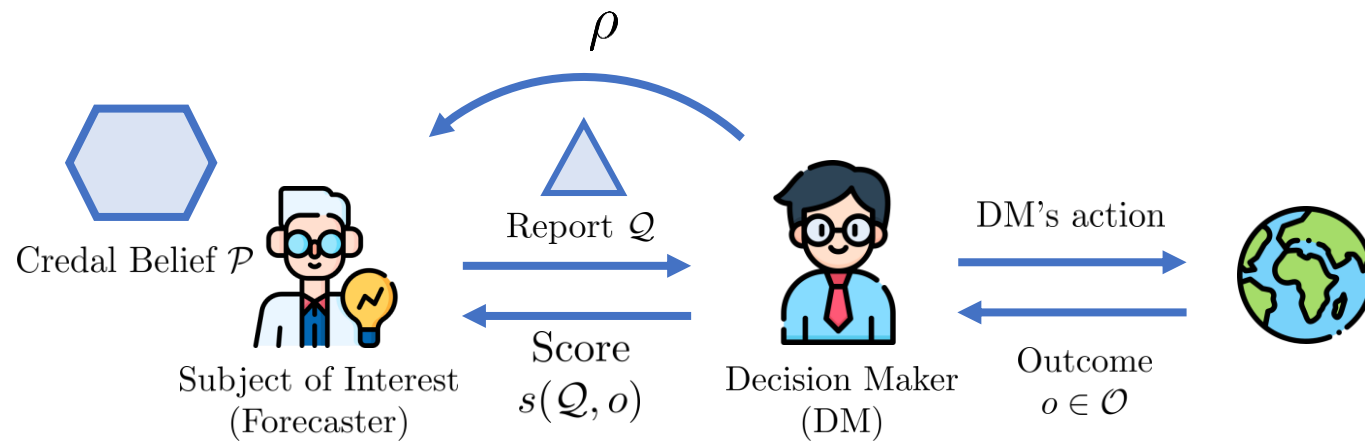
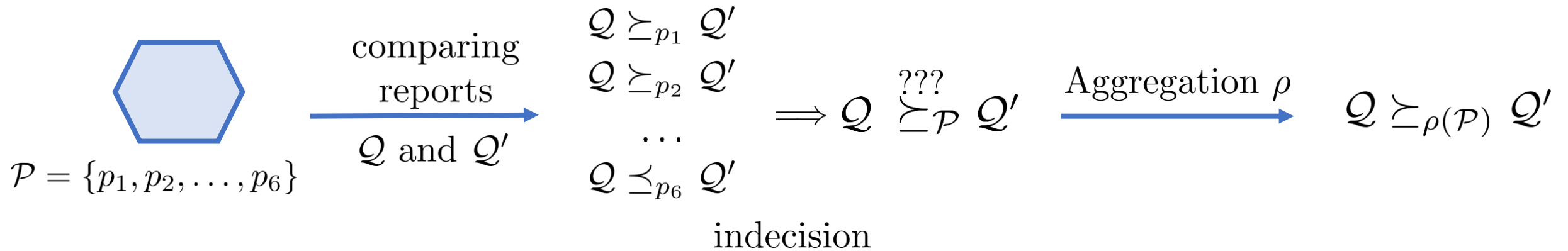
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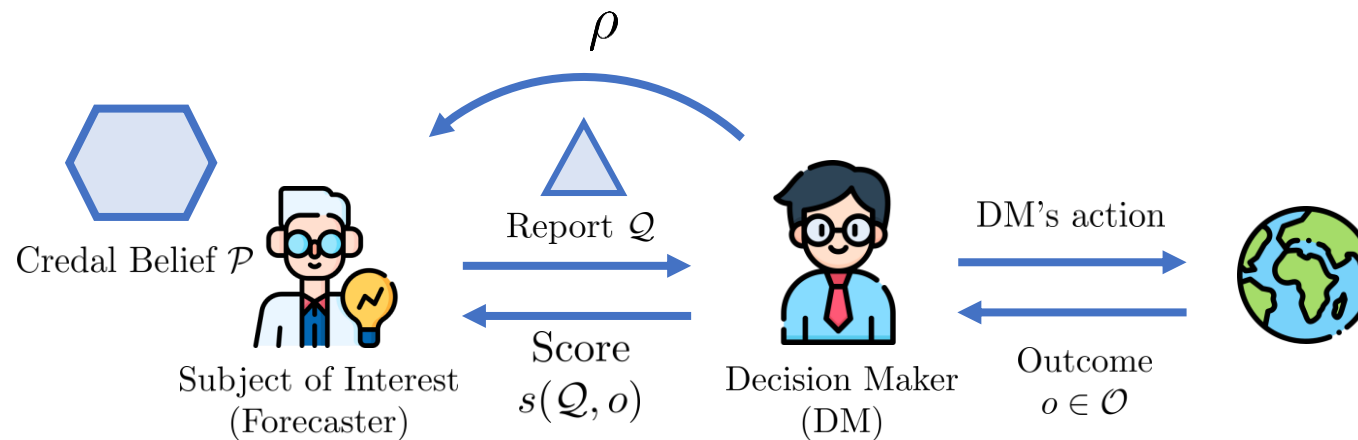
What is the exact form of this help?



IP Scoring Rules: How to score with a DM

$$s_{\rho}(\mathcal{Q}, o) = ku(a_{\mathcal{Q}, \rho}^*, o) + c \text{ where } k, c \in \mathbb{R}_{\geq 0} \text{ and } a_{\mathcal{Q}, \rho}^* = \arg \max_{a \in \mathcal{A}} \rho(\{\mathbb{E}_q[u(a, o)]\}_{q \in \mathcal{Q}})$$

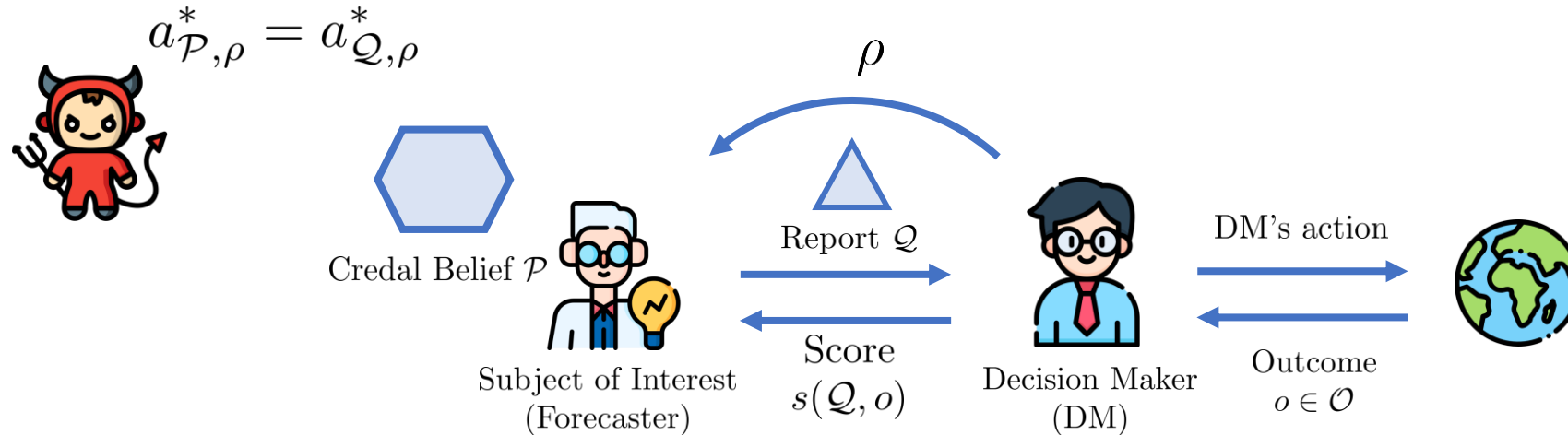
Proposition: s_{ρ} is proper for all ρ .



IP Scoring Rules: How to score with a DM

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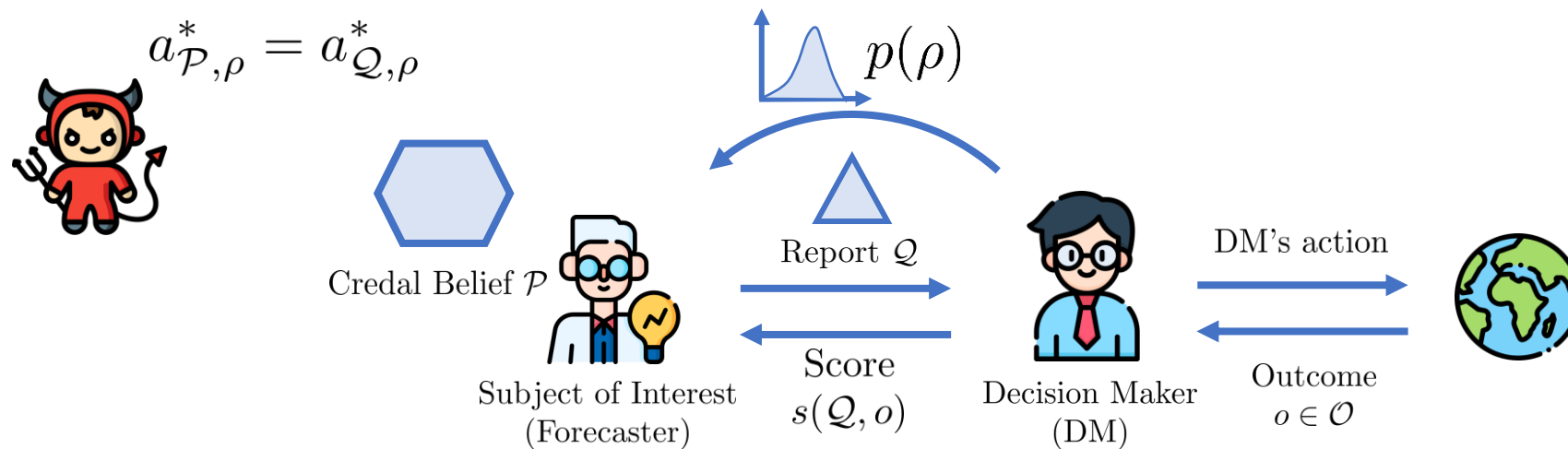
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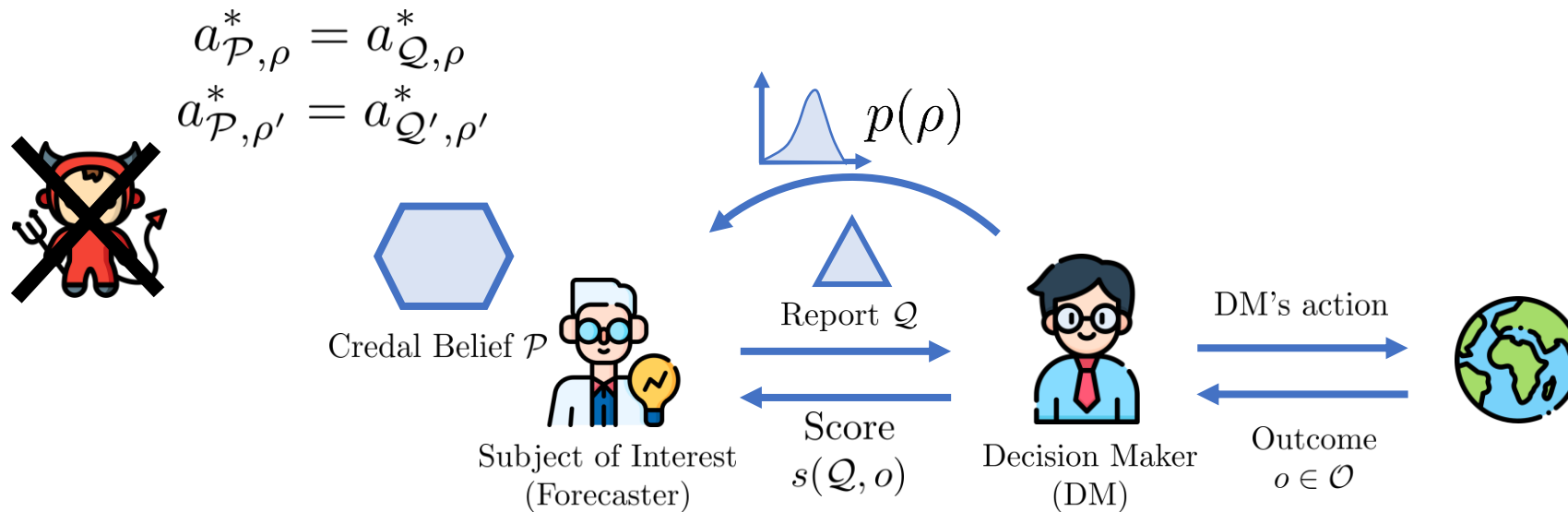
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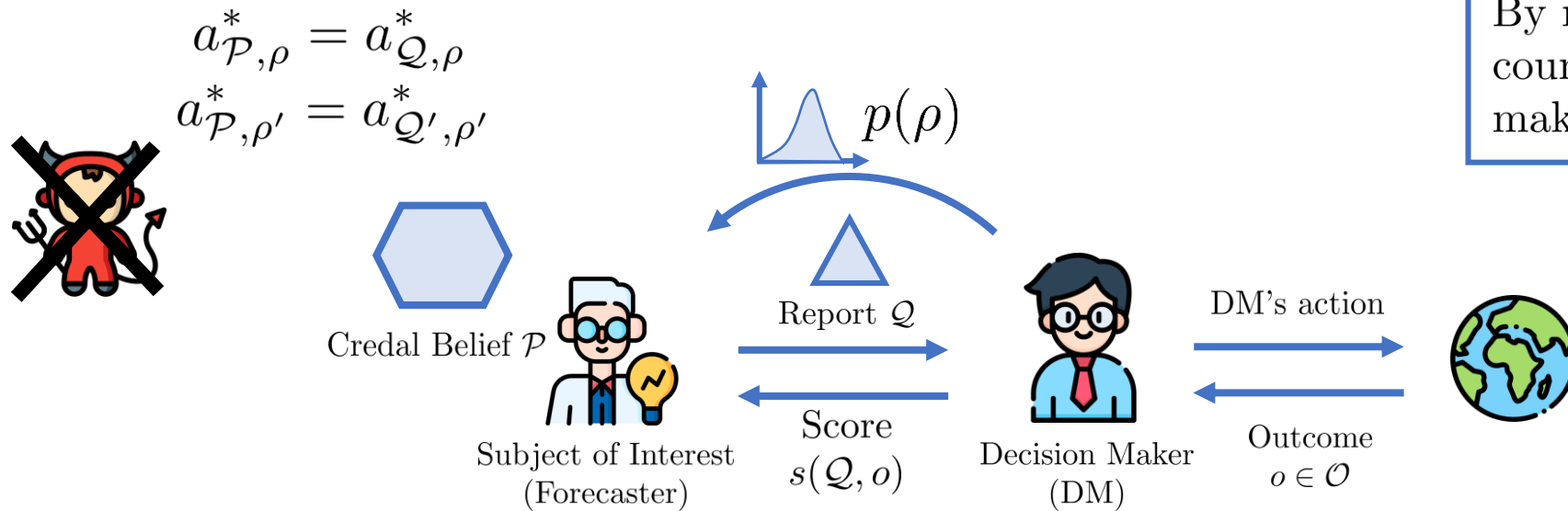


Strictly proper IP scoring rule

Theorem: s_ρ is strictly proper for $p(\rho)$ with full support.

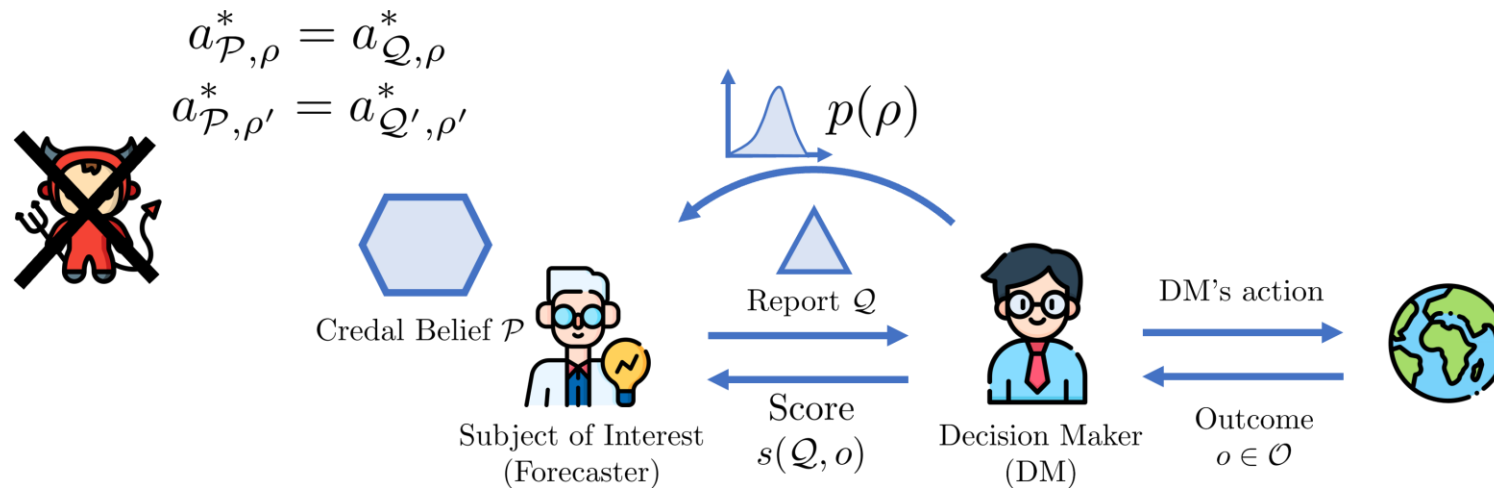
$$s_\rho(\mathcal{Q}, o) = \begin{cases} k_\rho u(a_{\rho, \mathcal{Q}}^*, o) + c_\rho & \text{if } p(\rho) > 0 \\ \Pi_o(\mathcal{Q}) & \text{if } p(\rho) = 0 \end{cases}.$$

By not telling which assignment counts for the final grade. I can make students do all of them.



Conclusion

1. Allow experts and algorithms to say “I don’t know exactly, but it’s between a and b”
2. Design of systems that explicitly embrace—not suppress—epistemic uncertainty.
3. Honest communication of uncertainty for trustworthy decisions.



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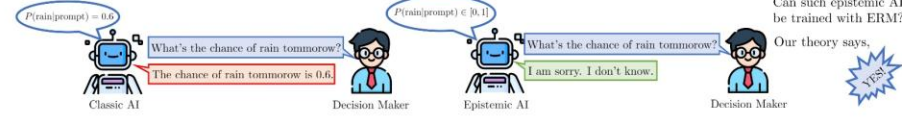
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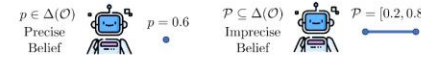


Our Motivation: Can we achieve epistemic AI with Empirical Risk Minimisation (ERM)?



1. Introduction

What's an Imprecise Forecaster? A forecaster is imprecise if their belief can be expressed as a set of distributions $\mathcal{P} \subseteq \Delta(\mathcal{O})$.



Credal Sets: A closed and convex set of probabilities $\mathcal{P} \subseteq \Delta(\mathcal{O})$ is called a credal set. For rational decision-making, imprecision in probability is equivalent to credal sets.



Scoring Rules: incentivise a forecaster to truthfully report their probability assessments.

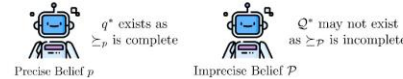
Precise Scoring Rule $s : \Delta(\mathcal{O}) \times \mathcal{O} \rightarrow \mathbb{R}$ Imprecise Scoring Rule $s : 2^{\Delta(\mathcal{O})} \times \mathcal{O} \rightarrow \mathbb{R}$

What does incentivising truthfulness mean? Let $\mathcal{P} \subseteq \Delta(\mathcal{O})$ be the true belief of an imprecise forecaster. A report $Q \subseteq \Delta(\mathcal{O})$ is truthful if $Q \simeq \mathcal{P}$. Where \simeq means equivalent credal sets for an imprecise forecaster.

Forecaster's Belief	Communication	Scoring Rule
Precise	-	Strictly Proper
-	-	Impossible
Imprecise	ρ	Proper
	$p(\rho)$	Strictly- Proper

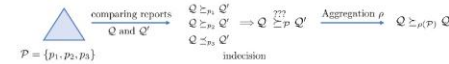
2. Why eliciting imprecise forecasts needs decision maker (DM)?

The forecaster needs help to complete the preference (\succeq_P) on reports.



Theorem: Naive extension of precise scoring rules to imprecise forecasts is impossible.

Aggregation Function: Combines multiple preferences into a single preference.



DM shares ρ with the forecaster



Tailored Scoring Rules: Allow us to parameterise the IP scoring rule $s_\rho : 2^{\Delta(\mathcal{O})} \rightarrow \mathbb{R}$ with ρ as share of DM's utility $u : \mathcal{A} \times \mathcal{O} \rightarrow \mathbb{R}$ over actions \mathcal{A} .

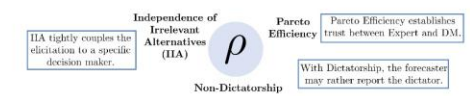
$$s_\rho(Q, o) = ku(a_\rho^*(Q, o)) + c \quad \text{where } k, c \in \mathbb{R}_{\geq 0} \quad \text{and } a_\rho^*(Q, o) = \arg \max_{a \in \mathcal{A}} \rho\{E_a[u(a, o)]\}_{Q \subseteq \Delta(\mathcal{O})}$$



Proposition: All imprecise scoring rules s_ρ are proper for any aggregation rule ρ .

3. Connection to Social Choice Theory

Axiomatisation of ρ When interpreting IP as a "collective" report of precise probabilities, a social choice perspective naturally emerges for the downstream DM.



Lemma: Let s_ρ be a tailored scoring rule. Then, the following holds:

- s_ρ is strictly proper for precise distributions if and only if $a_\rho^* := \arg \max_{a \in \mathcal{A}} E_a[u(a, o)]$ is a unique maximiser for all $q \in \Delta(\mathcal{O})$.
- s_ρ is not strictly proper, for any Pareto efficient ρ .

4. Characterisation of the strictly proper scoring rule

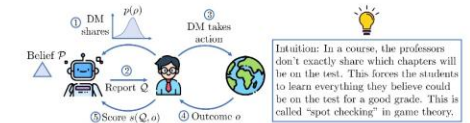
The DM shares a distribution $p(\rho)$ for truthful elicitation. Then the expected utility for forecast Q with belief \mathcal{P} is

$$V_{p(\rho)}^{\mathcal{P}}(Q) := E_{p(\rho)}[p\{E_a[s_\rho(Q, o)]\}_{a \in \mathcal{P}}]$$

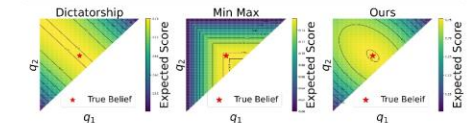
Strict Properness: A scoring rule is said to be strictly proper if for all \mathcal{P} , $V_{p(\rho)}^{\mathcal{P}}(P) > V_{p(\rho)}^{\mathcal{P}}(Q)$ for all Q such that $\mathcal{P} \neq Q$.

Main Theorem: (Strictly proper IP tailored scoring rules) An IP scoring rule s is strictly proper if $p(\rho)$ is a distribution with full support for the class of linear aggregations of ρ . Then for any $k_\rho, c_\rho \in \mathbb{R}_{\geq 0}$ and an arbitrary function $\Pi : 2^{\Delta(\mathcal{O})} \rightarrow \mathbb{R}$, the score is defined as

$$s_\rho(Q, o) = \begin{cases} k_\rho u(a_\rho^*(Q, o)) + c_\rho & \text{if } p(\rho) > 0 \\ \Pi_\rho(Q) & \text{if } p(\rho) = 0 \end{cases}$$



5. Simulation



Reporting the true belief uniquely maximizes the expected score. We conduct a simulation with a binary outcome (e.g., chance of rain tomorrow) for the true belief $\mathcal{P} = [0.4, 0.6]$. The forecaster reports an interval q_1, q_2 . For our implementation, we consider $\mathcal{A} = [0, 1]$ and $u(a, o) = (o - a)^2$

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